

1. Consider an LTI system with frequency resp

$$H(e^{j\omega}) = e^{-j(2\omega - \pi/4)} \left( \frac{1 + e^{-j\omega}}{1 + e^{j\omega}} \right)$$



Determine  $y[n]$ , the output of this system, if

$$x[n] = \cos\left(\frac{\pi n}{4}\right)$$

for all  $n$ . Your answer should contain a cos function.

$$\cos\left(\frac{9\pi n}{4}\right) = \frac{e^{j\frac{9\pi n}{4}} + e^{-j\frac{9\pi n}{4}}}{2}$$

$$\begin{aligned} H(e^{j\frac{\pi}{4}}) &= e^{-j(2(\frac{\pi}{4}) - \frac{\pi}{4})} \left( \frac{1 + e^{-j4(\frac{\pi}{4})} + 4e^{-j8(\frac{\pi}{4})}}{1 + 0.5e^{-j4(\frac{\pi}{4})}} \right) \\ &= e^{-j\frac{\pi}{4}} \left( \frac{1 + e^{-j\pi} + 4e^{-j2\pi}}{1 + 0.5e^{-j\pi}} \right) = e^{-j\frac{\pi}{4}} \left( \frac{1 + -1 + 4}{1 - 0.5} \right) \\ &= e^{-j\frac{\pi}{4}} \left( \frac{4}{1.5} \right) = 8e^{-j\frac{\pi}{4}} \end{aligned}$$

$$\begin{aligned} H(e^{-j\frac{\pi}{4}}) &= e^{-j(2(-\frac{\pi}{4}) - \frac{\pi}{4})} \left( \frac{1 + e^{-j4(-\frac{\pi}{4})} + 4e^{-j8(-\frac{\pi}{4})}}{1 + 0.5e^{-j4(-\frac{\pi}{4})}} \right) \\ &= e^{+j\frac{3\pi}{4}} \left( \frac{1 + -1 + 4}{1 - 0.5} \right) = 8e^{+j\frac{3\pi}{4}} \end{aligned}$$

$$y[n] = 8(e^{-j\frac{\pi}{4}} e^{j\frac{3\pi}{4}n} + e^{j\frac{3\pi}{4}} e^{-j\frac{\pi}{4}n})$$

$$= \frac{8(e^{-j\frac{\pi}{4}} e^{j\frac{\pi}{4}n} e^{j\frac{3\pi}{4}n} + e^{-j\frac{\pi}{4}} e^{j\frac{\pi}{4}n} e^{-j\frac{3\pi}{4}n})}{2}$$

$$y[n] = 8e^{+j\frac{\pi}{4}} \cos\left(\frac{9\pi}{4}n - \frac{\pi}{2}\right)$$